

Further Pure 1 - June 2006

① a) $\alpha + \beta = -\frac{1}{3} = 2$ $\alpha\beta = \frac{2}{3}$

b) i) using Pascal: $\alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$

ii) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
 $= 2^3 - 3\left(\frac{2}{3}\right)(2)$
 $= 8 - 4 = 4$

c) $\boxed{\text{sum}} = 4$

$\boxed{\text{product}} = \alpha^3 \times \beta^3 = (\alpha\beta)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$\rightarrow x^3 - \boxed{\text{sum}}x + \boxed{\text{product}} = 0$

$\rightarrow x^3 - 4x + \frac{8}{27} = 0$

$\rightarrow 27x^3 - 108x + 8 = 0$

② $y_{n+1} = y_n + h f(x_n)$

$x_1 = 2$

$y_1 = 3$

$h = 0.2$

$f(2) = \log_{10}(2) \approx$

$y_2 = 3 + 0.2 \times \log_{10}(2)$
 $= 3.060205\dots$

$x_2 = 2.2$

$y_2 = 3.060520\dots$

$h = 0.2$

$f(2.2) = \log_{10}(2.2)$

$y_3 = 3.0605\dots + 0.2 \times \log_{10}(2.2)$
 $= 3.128690\dots$
 $= 3.129 \text{ (3dp)}$

③ $\sum_1^n n^2 - n = \sum n^2 - \sum n$

$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1)$

$= \frac{1}{6}n(n+1) [(2n+1) - 3]$

Take out Factor of 2! $\rightarrow = \frac{1}{6}n(n+1) [2n-2]$

$= \frac{1}{3}n(n+1)(n-1)$

④ GS: $\theta = 2n\pi \pm \alpha$
 Key angle (α) = $\cos^{-1}(\sqrt{2}/2) = \pi/6$
 $\rightarrow 3x = 2n\pi \pm \pi/6$
 $\rightarrow x = 2/3 n\pi \pm \pi/18$

⑤ a) i) M^2

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

ii) $M^4 = M^2 \times M^2$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

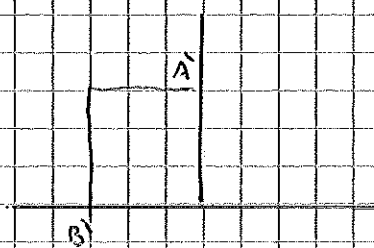
$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

b) In the form $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ $\theta = 45^\circ$
 Direction = clockwise
 \rightarrow Rotation 45° clockwise about $(0,0)$

c) $M^{2006} = M^{2000} \times M^6$
 $= [M^8]^{250} \times M^6$

M^8 must = I as it is 8 45° rotations
 $= I \times M^6$
 $= M^6 =$ rotation 270° clockwise
 or 90° anti-clockwise

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$



$$\textcircled{6} \text{ a) } (z+i)^* = (x+iy+i)^*$$

$$= x - iy - i$$

$$\text{b) } x - iy - i = 2i(x+iy) + 1$$

$$= 2xi - 2y + 1$$

REAL $x = -2y + 1$ $\textcircled{1}$

IMAG $-y - 1 = 2x$ $\textcircled{2}$

$$\textcircled{1} \times 2 \rightarrow 2x = -4y + 2$$

$$\text{sub } \textcircled{1} \text{ into } \textcircled{2} \rightarrow -y - 1 = -4y + 2$$

$$\rightarrow 3y = 3 \rightarrow y = 1$$

$$\text{From } \textcircled{1} \quad x = -2y + 1 \rightarrow x = -1$$

$$\rightarrow z = -1 + i$$

$$\textcircled{7} \text{ a) } y \text{ has been replaced by } (2y) \quad [\text{to give } 4y^2]$$

\rightarrow stretch, scale factor $1/2$ parallel to y -axis

$$\text{b) } x^2 - y^2 - 4x + 3 = 0$$

$$\rightarrow x^2 - 4x - y^2 + 3 = 0$$

$$\rightarrow (x-2)^2 - 4 - y^2 + 3 = 0$$

$$\rightarrow (x-2)^2 - y^2 = 1$$

\rightarrow Translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ or 2 units right in x direction

$$\textcircled{8} \text{ a) i) } f(1+h) = (1+h)^3 + (1+h)^2 - 1$$

$$= 1 + 3h + 3h^2 + h^3 + 1 + 2h + h^2 - 1$$

$$= h^3 + 4h^2 + 5h + 1$$

$$f(1) = 1 + 1 - 1 = 1$$

$$\therefore f(1+h) - f(1) = h^3 + 4h^2 + 5h + 1 - 1$$

$$= h^3 + 4h^2 + 5h$$

$$\text{ii) Gradient} = \frac{h^3 + 4h^2 + 5h}{h} = h^2 + 4h + 5$$

As $h \rightarrow 0$, gradient $\rightarrow 5$

b) i) At intersection: $x + 1 = \frac{1}{x^2}$

$\rightarrow x^3 + x^2 = 1$

$\rightarrow x^3 + x^2 - 1 = 0 = f(x)$

ii) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1$

$f(x) = x^3 + x^2 - 1$

$f'(x) = 3x^2 + 2x$

$x_2 = 1 - \frac{1}{5}$

$f(1) = 1 + 1 - 1 = 1$

$= \frac{4}{5}$

$f'(1) = 3 + 2 = 5$

c) $\int_1^{\infty} \frac{1}{x^2} = \int_1^m x^{-2} = \left[-x^{-1} \right]_1^m$

$= -\frac{1}{m} - (-1) = 1 - \frac{1}{m}$

As $m \rightarrow \infty$, $\frac{1}{m} \rightarrow 0$

$\int \rightarrow 1 = \text{Area of Region}$

(9) a) i) Intersects x-axis when $y = 0$

$\rightarrow 0 = \frac{(x+1)(x-3)}{(x)(x-2)}$

$\rightarrow 0 = (x+1)(x-3) \rightarrow x = -1 \text{ or } x = 3$

$\rightarrow (-1, 0) \text{ or } (3, 0)$

ii) A Denominator = 0 when $x = 0$ and $x = 2$

As $x \rightarrow \infty$, $y \rightarrow \frac{x^2}{x^2} \rightarrow 1$

\therefore Asymptotes: $x = 0$, $x = 2$, $y = 1$

b) i) $k = \frac{(x+1)(x-3)}{x(x-2)}$

$\rightarrow kx(x-2) = (x+1)(x-3)$

$\rightarrow kx^2 - 2kx = x^2 - 2x - 3$

$\rightarrow kx^2 - x^2 - 2kx + 2x + 3 = 0$

$\rightarrow (k-1)x^2 + (2k-2)x + 3 = 0$

For intersection, $b^2 - 4ac \geq 0$

$$\rightarrow (2k-2)^2 - 4(k-1) \times 3 \geq 0$$

$$\rightarrow 4k^2 - 8k + 4 - 12k + 12 \geq 0$$

$$\rightarrow 4k^2 - 20k + 16 \geq 0$$

$$\rightarrow k^2 - 5k + 4 \geq 0$$

$$\rightarrow (k-1)(k-4) \geq 0$$

ii) At st point, $b^2 - 4ac = 0$

$$\rightarrow (k-1)(k-4) = 0$$

$$\rightarrow k = 1 \quad \text{or} \quad k = 4$$

$$\swarrow$$

$$(1-1)x^2 - (2-2)x + 3 = 0$$

$$\rightarrow 3 = 0$$

NO SOLUTION

$$\searrow$$

$$(4-1)x^2 - (8-2)x + 3 = 0$$

$$\rightarrow 3x^2 - 6x + 3 = 0$$

$$\rightarrow x^2 - 2x + 1 = 0$$

$$\rightarrow (x-1)(x-1) = 0$$

$$\downarrow$$

$$x = 1, y = 4$$

$$= (1, 4)$$

c)

